NAG Toolbox for MATLAB

s17al

1   Purpose

s17al determines the leading **n** zeros of one of the Bessel functions $J_\alpha(x)$, $Y_\alpha(x)$, $J'_\alpha(x)$ or $Y'_\alpha(x)$ for real x and nonnegative α.

2   Syntax

```
[x, ifail] = s17al(a, n, mode, 'rel', rel)
```

3   Description

s17al attempts to find the leading N zeros of one of the Bessel functions $J_\alpha(x)$, $Y_\alpha(x)$, $J'_\alpha(x)$ or $Y'_\alpha(x)$, where x is real. When α is real, these functions each have an infinite number of real zeros, all of which are simple with the possible exception of $x = 0$. If $\alpha \geq 0$, the nth positive zero is denoted by $j_{\alpha,n}, j'_{\alpha,n}, y_{\alpha,n}$ and $y'_{\alpha,n}$, respectively, for $n = 1, 2, \ldots, N$, except that $x = 0$ is counted as the first zero of $J'_\alpha(x)$ when $\alpha = 0$. Since $J'_0(x) = -J_1(x)$, it therefore follows that $j'_{0,1} = 0$ and $j'_{0,n} = -j_{1,n-1}$ for $n = 2, 3, \ldots, N - 1$. Further details can be found in Section 9.5 of Abramowitz and Stegun 1972.

s17al is based on Algol 60 procedures given by Temme 1979. Initial approximations to the zeros are computed from asymptotic expansions. These are then improved by higher-order Newton iteration making use of the differential equation for the Bessel functions.

4   References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Temme N M 1976 On the numerical evaluation of the ordinary Bessel function of the second kind *J. Comput. Phys.* **21** 343–350

Temme N M 1979 An algorithm with Algol 60 program for the computation of the zeros of ordinary Bessel functions and those of their derivatives *J. Comput. Phys.* **32** 270–279

5   Parameters

5.1   Compulsory Input Parameters

1:    **a – double scalar**

The order α of the function.

Constraint: $0.0 \leq$ **a** ≤ 100000.0.

2:    **n – int32 scalar**

the number N of zeros required.

Constraint: **n** ≥ 1.

3:    **mode – int32 scalar**

Specifies the form of the function whose zeros are required.

mode $= 1$

The zeros of $J_\alpha(x)$ are required.

mode $= 2$

The zeros of $Y_\alpha(x)$ are required;

mode $= 3$

The zeros of $J'_\alpha(x)$ are required;

mode $= 4$

The zeros of $Y'_\alpha(x)$ are required.

Constraint: $1 \le$ **mode** ≤ 4.

5.2 Optional Input Parameters

1: **rel – double scalar**

The relative accuracy to which the zeros are required.

Suggested value: the square root of the ***machine precision***.

Default: $\sqrt{}$

Constraint: **rel** > 0.0.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **x(n) – double array**

The N required zeros of the function specified by **mode**.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **a** < 0.0,
or   **a** > 100000.0,
or   **n** ≤ 0,
or   **mode** < 1,
or   **mode** > 4,
or   **rel** ≤ 0.0.

7 Accuracy

If the value of **rel** is set to 10^{-d}, then the required zeros should have approximately d correct significant digits.

8 Further Comments

None.

9 Example

```
a = 0;
n = int32(5);
mode = int32(1);
[x, ifail] = s17al(a, n, mode)

x =
    2.4048
    5.5201
    8.6537
   11.7915
   14.9309
ifail =
        0
```